Retailers and consumers in sequential auctions of collectibles

Stéphanie Vincent Lyk-Jensen *Danish Transport Research Institute*
Olivier Chanel *Centre National de la Recherche Scientifique, GREQAM, Marseille*

Abstract. We analyse an independent private-value model, where heterogeneous bidders compete for objects sold in sequential second-price auctions. In this heterogeneous game, bidders may have differently distributed valuations, and some have multi-unit demand with decreasing marginal values (retailers); others have a specific single-unit demand (consumers). By examining equilibrium bidding strategies and price sequences, we show that the presence of consumers leads to more aggressive bidding from the retailers on average and heterogeneous bidders is a plausible explanation of the price decline effect. The study of the expected revenue of the seller confirms the interest of auctioneers in inviting different types of bidders. JEL classification: D44

Détaillants et consommateurs dans les enchères séquentielles d'objets de collection. Nous analysons un modèle à valeurs privées indépendantes dans lequel des enchérisseurs hétérogènes sont en concurrence pour des objets vendus lors d'enchères séquentielles au second prix. Dans ce jeu hètèrogène, les enchérisseurs tirent leurs évaluations de distributions qui peuvent différer, et certains (les revendeurs) ont des demandes multi unitaires avec des évaluations marginales décroissantes alors que les autres (les consommateurs) ont une demande individuelle spécifique. L'examen des stratégies d'équilibre et des séquences de prix montrent que la présence de consommateurs conduit en moyenne a des enchères plus agressives de la part des revendeurs, et que la présence d'enchérisseurs hétérogènes constitue une explication plausible a la décroissance des prix. L'étude de l'espérance du revenu du vendeur confirme l'intérêt des commissaires-priseurs à inviter différentes catégories d'acheteurs.

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1. Introduction

Several types of bidders are often observed at auctions of art objects (sculptures, paintings, antiques, wines, and other collectibles), cars, and dwellings and in most auctions where different bidders have different average valuations or/and different demand schedules. Casual observations of jewellery auctions at Crédit Municipal (CM)[1] indicate at least two types of bidders: retailers seeking resale opportunities and consumers who are more subject to buying on whim.

Simple statistics on the data[2] show that consumers account for 93% of the buyers and buy 57% of the jewellery on auction. Most of them (i.e., 62%) limit their purchases to one object. When they buy more than one piece of jewellery, they usually buy different types: one ring and one watch, for example. Retailers account for 7% of the buyers and buy 43% of the jewellery on auction.[3] They seem to specialize in different types of jewellery: high-quality rings, low-quality jewellery or watches, for example. They buy more than one object and, as their respective resale markets are distinct, are in competition with other retailers only at the auction. Indeed, discussions with auctioneers reveal that retailers have both geographically separate markets and different kinds of customers on their resale market.

The modelling of such markets requires a multi-object auction framework allowing for heterogeneous bidders. However, most of the auction literature has focused on single-object auctions where bidders are homogeneous. In the case of multi-object auctions, Weber (1983) and Milgrom and Weber (2000) provide a first overview of the issues in modelling multi-object auctions with asymmetric bidders but where individual bidder demand is restricted to one unit. Papers allowing for bidders who purchase more than one unit are usually restricted to homogeneous bidders (Black and de Meza 1992; Katzman 1999). In multi-unit auctions, heterogeneity of bidders has been modelled as different participation costs (see von der Fehr 1994), as asymmetric information in common-value models (see Engelbrecht-Wiggans and Weber 1993), or as differences in the demand size of the bidders (see Krishna and Rosenthal 1996; Rosenthal and Wang 1996; Burguet and Sakovics 1997; Branco 1997; and Menezes and Monteiro 2004).

The price trend constitutes another important feature in the study of sequential auctions. In fact, the properties of equilibrium price sequences are more important than may appear, since the theory can be tested by observation of price sequences from real-world auctions. This theory holds that sequential auctions of identical objects should result on average in identical (Weber 1983, for independent private

1 CM is a modern pawnshop, which lends some 30% to 50% of the estimated value of items deposited by borrowers. If the owner defaults, CM sells the object at an English auction, that is, an ascending oral auction, and returns any surplus to the borrower. This is a special type of auction, with a captive seller who has no strategic freedom with respect to the sale (no influence through a reserve price).

2 The database records the sales of jewellery at 27 judicial English public auctions between April 1994 and September 1996 at Crédit Municipal de Marseille. The database contains 3,157 observations, with the following information for each: the expert evaluation, the selling price, the characteristics of the piece of jewellery, and the identity of the buyers: retailers or consumers.

3 In fact, eight of the retailers (1% of the buyers) buy 25% of the jewellery on auction.

values) or rising prices (Milgrom and Weber 2000, for affiliated values). However, many empirical studies have found evidence of declining prices in auctions of wine (Ashenfelter 1989; McAfee and Vincent 1993; Ginsburgh 1998), condominium units (Ashenfelter and Genesove 1992), jewellery (Chanel, Gérard-Varet, and Vincent 1996), works of art (Pesando and Shum 1996; Beggs and Graddy 1997), flowers (Van den Berg, Van Ours, and Pradhan 2001), fish (Pezanis-Christou 1997) and so forth.

If prices show a regular pattern, it is crucial to determine if it is an anomaly, as postulated by Ashenfelter (1989), or if it can be explained by market characteristics and hence by the rational behaviour of bidders.[4] Cassady (1967) observes that auction prices are the result of competition among bidders who have different 'intensities of desire' in accordance with their valuations. The price trend therefore seems to rely on the identities and characteristics of the bidders. In a previous study of jewellery auctions at Crédit Municipal, Chanel, Gérard-Varet, and Vincent (1996) find that price trends differ according to the type of jewellery. They suggest that the fact that two types of bidders (consumers and retailers) coexist in this market may explain the price patterns.

The purpose of this paper is to focus on sequential asymmetric multi-object auctions and to study the effect of the presence of heterogeneous bidders on equilibrium strategies, on price trends, and on the revenue of the seller. Bidders are heterogeneous in the sense that some bidders (referred to as retailers) have multi-unit demand and others (referred to as consumers) have single-unit demand specific to a particular unit. In this framework, the Revenue Equivalence Principle no longer holds. Since the type of objects considered here is usually sold through English auction, we consider a two-stage second-price auction, since we show the revenue equivalence between these formats.

Contrary to the multi-unit models of Ortega Reichert (1968), Hausch (1986), and Caillaud and Mezzetti (2004), we focus on a model where bidders' valuations are independent but ranked in decreasing order, so that a bidder who wins a given auction is not concerned about revealing his valuation. In the present paper, each retailer ranks his independently drawn valuations in descending order. Retailers have more than one signal because they rely on different consumers' having different tastes or willingness to pay for a specific object, and they are therefore interested in both objects. Consumer demand is specific to a particular unit: consumers interested in the first object will not attend the second auction, regardless of whether they win or lose the first auction.

4 McAfee and Vincent (1993) show that, if bidders have non-decreasing absolute risk aversion, an expectation of declining price results. But other contributions have shown that even with risk-neutral bidders, it is possible to observe decreasing price sequences. Other explanations may be sale in decreasing order of quality (Pesando and Shum 1996; Beggs and Graddy 1997); the rules and format of auction, for example, a pooled auction (Gale and Hausch 1994); the existence of a buyer's option or endogeneous uncertainty regarding the supply (Black and de Meza 1992; Burguet and Sakovics 1994); exogeneous uncertainty regarding the supply, in fish auctions, for example (Pézanis-Christou 1997); the presence of absentee bidders (Ginsburgh 1998); participation costs (von der Fehr 1994); a decreasing valuation of objects according to their position in the auction (Kittsteiner, Nikutta, and Winter 2004), and so on.

The main results of this paper are the following. The presence of bidders with specific single-unit demand leads on average to more aggressive bidding from the retailer. This retailer may be considered the winner of a pre-auction between retailers.[5] Based on theoretical models, decreasing price patterns can be observed when a retailer and consumers compete simultaneously, thus corroborating the empirical findings from various auctions, particularly jewellery auctions. The revenue of the seller is found to be greater for the heterogeneous game than for the homogeneous games under reasonable assumptions. Moreover, the second-price sealed-bid auction and the English auction are still revenue equivalent where there is only one retailer and any number of consumers, thus leading to an efficient outcome.

The paper is organized as follows. Section 2 presents the assumptions behind the games. Section 3 refers to results previously established by the literature for homogeneous bidders. Section 4 analyses the case of heterogeneous bidders. Section 5 studies how heterogeneity influences the equilibrium, the price pattern, and the expected revenue of the seller compared with homogeneous bidders games. Section 6 concludes the paper. All proofs are contained in the appendices.

2. Assumptions

We consider a sequential second-price sealed-bid auction: bidders submit closed bids and the winner pays the bid of the second highest bidder. Two types of bidders participate in a sequence of two auctions: retailers and consumers. Each retailer i has a pair of valuations $w_i = (w_i^H, w_i^L)$, where w_i^H represents the value he attributes to the first unit he buys and w_i^L is the value for the second unit he buys. We assume that retailers have decreasing marginal utility. They independently draw two valuations from a distribution G, and these two valuations are ranked so that $w_i^H > w_i^L$.[6] The $k + 1$ retailers have independent resale markets – and consequently different and privately known resale values – and multi-unit demand. They thus participate in both auctions whatever the result of the first one. The valuations w_i^H and w_i^L are order statistics: the cumulative distribution of w_i^H is $G^2(\cdot)$, while the cumulative of w_i^L is $G(\cdot)[2 - G(\cdot)]$. Finally, the conditional distribution of w_i^L given w_i^H, is $G(\cdot)/G(w_i^H)$.

The second category of bidders is c consumers, who buy without having resale opportunities. Their demand is specific to a particular unit. If consumers are interested in the first object, they will not participate in the second auction, whether they win or lose the first one. Likewise, consumers interested in the second object

5 We have personally witnessed, by chance, this type of bidding ring between jewellery retailers in a restaurant just before an auction. Extending this case to several retailers would be very complex (ex ante and ex post asymmetries, conditional information, etc.).

6 The intuition behind this assumption is that retailers know the exact resale value of the object and rank their valuations in descending order with a view to exploiting their most lucrative business opportunities first. This can be explained by the fact that retailers have to satisfy different customers' tastes or fill orders from their clients.

will not participate in the first auction. Consumer valuations are independent draws from a distribution F. Let c_1 and c_2 be the number of consumers participating in the first and the second auction.[7]

The two objects being sold are considered substitutes by the retailers, but their aesthetic differences (or features) make them unique for the consumers, for example, two gold necklaces of the same weight but with different links or two otherwise identical dwellings located on different floors. This also applies to auctions of used cars and other collectibles.

F and G are strictly increasing and continuously differentiable on $[\underline{v}, \bar{v}]$, $\underline{v} < \bar{v}$, with $F(\underline{v}) = G(\underline{v}) = 0$ and $F(\bar{v}) = G(\bar{v}) = 1$. Moreover, it can be assumed that consumers have, on average, higher valuations than retailers; for example, F (first order) stochastically dominates G.

All the bidders have independent private values, are risk neutral, and their identities are common knowledge prior to the auction.

The paper compares two games: the homogeneous retailer game, where $(k+1)$ retailers compete in the first and second auction; and the heterogeneous game, where a retailer competes against c_1 consumers in the first auction and c_2 consumers in the second auction.[8]

3. Solving the homogeneous retailer game

Black and de Meza (1992) were the first to derive equilibrium strategies in a sequence of two second-price auctions when retailers have a multi-unit demand. However, Katzman (1999) showed that this type of equilibria, where retailers bid for their lowest valuations at the first auction, is not robust to an increase in the number of bidders. We restate below the game solved by Katzman (1999) and its main results and intuition.

In the second and last auction, the (weakly) dominant strategy is to bid one's valuation. Hence, retailer i bids w_i^L if he has won the first auction and w_i^H otherwise.

In the first auction, the optimal bid is obtained when a retailer i is indifferent to whether he wins the current auction or the next one. Under the symmetric assumption, the optimal bid can be written as follows:

$$b^K\left(w_i^H\right) = E\left[\left. W_{2-i}^{(2k)} \right| W_{1-i}^{(2k)} = w_i^H\right], \tag{1}$$

where $W_{2-i}^{(2k)}$ (or $W_{1-i}^{(2k)}$) denotes the second-highest (or the highest) valuation among the $2k$ valuations of the other bidders. Hence, $W_{2-i}^{(2k)}$ is the second-order statistic of $2k$ independent draws from a distribution G, whose expected value,

7 The number of consumer bidders taking part in each auction is common knowledge.

8 A third game, where only consumers compete, will also enter the comparison. This homogeneous consumer game is simple, consisting of two second-price sealed-bid auctions, where the dominant strategy consists in bidding the true valuation.

conditional on the first-order statistic being equal to $W_{1-i}^{(2k)}$, is

$$b^K\left(w_i^H\right) = \int_{\underline{v}}^{w_i^H} s\frac{dG^{2k-1}(s)}{G^{2k-1}\left(w_i^H\right)}$$
$$= w_i^H - \int_{\underline{v}}^{w_i^H} \frac{G^{2k-1}(s)}{G^{2k-1}\left(w_i^H\right)}ds. \tag{2}$$

Retailers shade below their high valuation, and the higher the w_i^H, the higher the shading effect will be. The monotonicity of G implies that this shading approaches 0 as the number of retailers increases. Katzman (1999) established that, in expectation, the winning price is increasing on average.

4. Solving the heterogeneous game

4.1. Bidding strategy

The heterogeneous game is an original game. A retailer participates in both auctions and competes against c_1 consumers in the first auction and c_2 consumers in the second one. The following constitutes an equilibrium of the sequential auction.

PROPOSITION 1. *In two-unit sequential second-price auctions with one retailer and c_1 and c_2 consumers, the following is an equilibrium:*

- ⋆ *the consumers bid their valuation at each auction;*
- ⋆ *the retailer with valuations $w = (w^H, w^L)$ bids as follows:*
  - *at the first auction $b^B(w^H, w^L) = w^H - \int_{w^L}^{w^H} F^{c_2}(s)ds$;*
  - *at the second auction* $\begin{cases} w^L, & \text{if he won the first auction,} \\ w^H, & \text{if he lost the first auction.} \end{cases}$

Note that b^B can also be written as $w^H - (\pi_\ell^B - \pi_w^B)$, where π_w^B is the retailer's expected profit in the second auction if he won the first auction and π_ℓ^B is the retailer's expected profit in the second auction if he lost the first one (see appendix A for details). This can be interpreted as 'the retailer's bid in the first auction is equal to his highest valuation (w^H) less a premium equal to the difference in the values attached to losing and winning, i.e., the opportunity cost of winning the first auction.'

REMARK 1. *The equilibrium bid function b^B belongs to the interval $[w^L, w^H]$, is strictly increasing, and generates bids shaded below the high valuation.*

REMARK 2. *The fact that the retailer first bid is strictly increasing in high valuations, coupled with the bidding of valuations in both auction for consumers and in the second auction for the retailer, indicates that this equilibrium results in an efficient outcome.*

REMARK 3. *Under the assumptions of the game, there is a revenue equivalence between the second-price sealed-bid auction and the English auction when there is* *one retailer and any number of consumers. Indeed, the same bidder will win the same object and the lowest-bidding type makes zero expected profit in both types of auction.*

4.2. *Price trend*

The higher the number of consumers participating in the second auction, the lower the retailer will shade his high valuation w^H, so that b^B approaches w^H when c_2 is large enough. Notice that the number of consumers in the first auction does not affect the retailers' strategy but will change $F^{c_1}(b^B)$, his probability of winning the first auction. Consequently, both c_1 and c_2 influence the prices observed in the first and the second auction and, hence, the price trend.

RESULT 1. *The expected price trend has the sign of the following expression:*

$$\int_{\underline{v}}^{\bar{v}} \int_{\underline{v}}^{w^H} \left\{ \begin{array}{l} -F^{c_1}(b^B) \int_{w^L}^{w^H} E'(s, c_2)ds - \int_{\underline{v}}^{b^B} E'(s, c_1)ds \\ + \int_{\underline{v}}^{w^H} E'(s, c_2)ds + E(\underline{v}, c_2) - E(\underline{v}, c_1) \end{array} \right\} dG(w^L)dG(w^H) \quad (3)$$

with $E'(s, N) \equiv NF^{N-1}(s)[1 - F(s)]$
and $E(\underline{v}, N) \equiv N\int_{\underline{v}}^{\bar{v}} s[1 - F(s)]\,dF^{N-1}(s)$.

The direction of the trend (only) depends on the shape of F and the number of consumers. However, a price decline is obtained for a large class of functions.

EXAMPLE 1. *Consider the class of distribution functions defined (on the unit interval) by s^α for F, s^β for G with $\alpha, \beta > 0$. F first-order stochastically dominates G when $\alpha > \beta$. Applying formula (3) in the case $c_1 = c_2 = N$ leads to:*

$$N\beta^2 \int_0^1 \int_0^{w^H} \left\{ \begin{array}{l} (b^B)^{\alpha N} \int_{w^L}^{w^H} [s^{\alpha N} - s^{\alpha N - \alpha}]ds \\ - \int_{b^B}^{w^H} [s^{\alpha N} - s^{\alpha N - \alpha}]ds \end{array} \right\} (w^L)^{\beta-1}(w^H)^{\beta-1}dw^L dw^H. \quad (4)$$

This expression is always non-positive, as can be shown by numerical integration. The price trend thus is always decreasing and tends to constancy when N increases.

5. Comparing the two games

The number of opponent bidders in each auction and in each game is set to N (i.e., $c_1 = c_2 = k = N$).

5.1. *The strategy used by the retailer in the first auction*

To compare the retailer bid in the first auction when competing with retailers versus consumers, we need to compare b^B and b^K for $G(s) \geq F(s)$ for all $s \in [\underline{v}, \bar{v}]$.

The retailer is more aggressive; that is, he makes a higher bid against consumers than against other retailers if, for every (w^L, w^H) pair,

$$b^K \leq b^B \Leftrightarrow \int_{w^L}^{w^H} F^N(s)\, ds \leq \int_{\underline{v}}^{w^H} \frac{G^{2N-1}(s)}{G^{2N-1}(w^H)} ds. \qquad (5)$$

This is obviously the case when $N = 1$ for any distribution compatible with the assumptions of section 2. However, the inequality may or may not hold when $N > 1$, depending on the pair (w^L, w^H). Hence, we compute the expected first auction bid and obtain the following results.

RESULT 2. *The expected first-auction bid of a retailer playing against respectively* c_1 *and* c_2 *consumers is* $E[b^B] = \int_{\underline{v}}^{\bar{v}} s dG^2(s) - 2 \int_{\underline{v}}^{\bar{v}} F^{c_2}(s)\, G(s)\, [1 - G(s)]\, ds.$

RESULT 3. *The expected first-auction bid of a retailer playing against k other retailers is:* $E[b^K] = \int_{\underline{v}}^{\bar{v}} s dG^2(s) - [2/(2k-3)] \int_{\underline{v}}^{\bar{v}} [G^2(s) - G^{2k-1}(s)] ds.$

RESULT 4. *In the first auction,* a *retailer will always bid on average more aggressively against N opponent consumers than against N opponent retailers for* $G \geq F$.

The intuition behind result 4 is as follows. The retailer's opportunity cost of winning the first auction is higher when he is facing retailer opponents than when he faces consumers. Indeed, the expected bid of one opponent retailer decreases from a high value to a low value if he loses the first auction, while expected bids by opponent consumers in the second auction are unaffected by the result of the first auction: they always bid their value. Given this, a retailer will have less incentive to bid high in the first auction when he is facing another retailer, which is stated in result 4.

5.2. The price trend

Sequential auction models theoretically predict price trends in terms of how p^1 relates to p^2. The Ratio of the Sum of Prices (RSP) defined as $E[p^2]/E[p^1]$ is then used as estimator.[9]

Because no general results could be drawn from formula (3), we consider the class of distribution of example 1. Figure 1 presents results for the heterogeneous game with $\beta = 1$, for different levels $\alpha \geq 1$ of stochastic dominance for F and for different numbers of opponents.

We observe a decreasing price trend, but the magnitude of this decrease diminishes when the degree of stochastic dominance increases. This is due to the

9 The two other widely used estimators are the Arithmetic Mean of Ratios, defined as $AMR = (1/n) \sum_{\ell=1}^{n} (P_\ell^2 / P_\ell^1)$, and the Geometric Mean of Ratios, defined as $GMR = [\prod_{\ell=1}^{n} (P_\ell^2 / P_\ell^1)]^{\frac{1}{n}}$, with P_ℓ^1 and P_ℓ^2 strictly positive. Contrary to RSP, these two estimators mask the price levels by using price ratios in the formula (see Chanel and Vincent 2004 on the computation of price trends in sequential auctions).

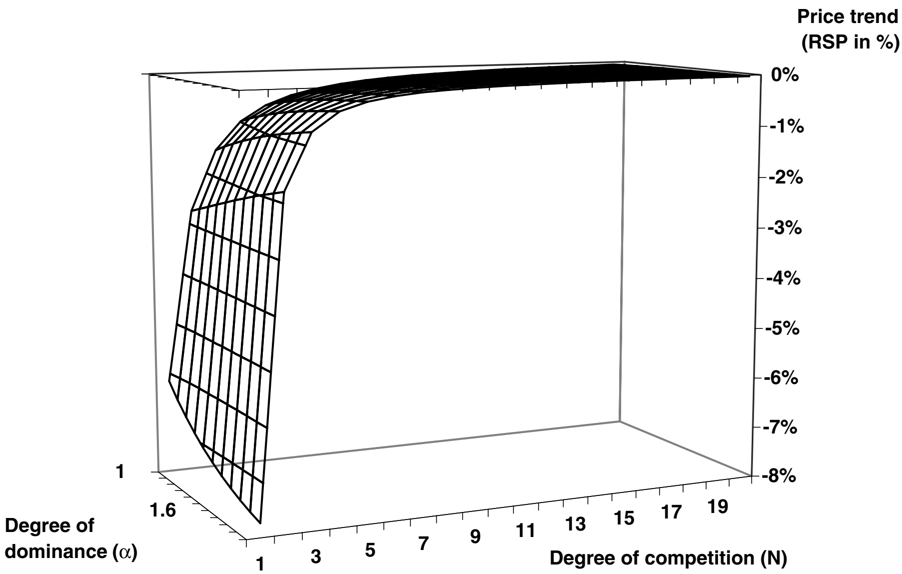

FIGURE 1 Price trend according to the degree of stochastic dominance and of competition (RSP)

fact that, as α increases, the retailer's probability of winning and the shading effect diminish. The trend also approaches constancy, for the same reasons, as the number of opponents increases. For $c_1 = c_2$, the price trend is constant for the homogeneous consumer game. Note that the two auctions are independent: consumers participating in the first auction will not participate in the second auction. The price trend is increasing for the homogeneous retailer game but approaches constancy when the number of opponents increases, since the probability of the same bidder winning both auctions decreases. (See appendix B for further details).

5.3. The expected revenue of the seller

The expected revenue of the seller $E(p^1) + E(p^2)$ in the case $G = s$ and $F = s^\alpha$, with $\alpha \geq 1$, is computed for the three games. Intuitively, two elements interact to explain the relative ranking of the games: the retailer's first auction bid shading effect and the fact that retailers have stochastic order valuations. Figure 2 presents the main findings. It depends on the degree of competition and the level of stochastic dominance.

5.3.1. When competition is low ($N = 1$, i.e., two bidders in each auction)

The homogeneous retailer game always gives the lowest expected revenue and the heterogeneous game leads to the highest expected revenue in the absence of stochastic dominance. When $N = 1$, the very high shading effect in the

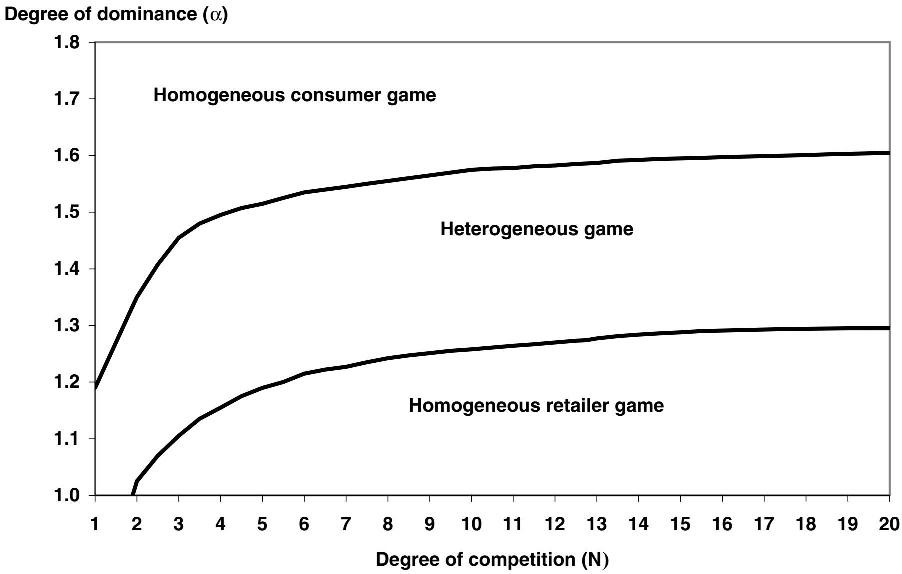

FIGURE 2 Game that maximizes the expected revenue of the seller according to the degree of stochastic dominance and of competition

homogenous retailer game overrides the impact of stochastic order valuations: the homogenous consumer game leads to a higher expected revenue for the seller. In the heterogeneous game, the shading effect is lower than in the homogeneous retailer game and is overridden by the impact of having stochastic order valuations: the heterogeneous game provides a higher expected revenue than the homogeneous consumer game. When consumers have higher valuations than retailers, the homogeneous consumer game leads to a higher expected revenue for a sufficiently high degree of stochastic dominance ($\alpha > 1.2$).

5.3.2. When competition increases in each auction ($N > 1$)

In the absence of stochastic dominance, the homogeneous retailer game always leads to higher expected revenue for the seller. Indeed, the shading effect of the retailer becomes negligible as N increases. All bidders have stochastic order valuations, contrary to the consumer homogeneous game, and overall competition increases, since the retailers attend both auctions. However, if consumers have higher valuations than retailers, the picture is more complex, as we face a trade-off between stochastic dominance and number of opponents (see figure 2):

- For high levels of stochastic dominance ($\alpha > 1.6$), the homogeneous consumer game always leads to a higher expected revenue for the seller.

- For combinations of weak or mild competition and moderate stochastic dominance ($1.5 > \alpha > 1.05$), the heterogeneous game leads to the highest expected revenue.
- For combinations of stronger competition and low or mild stochastic dominance ($1.2 > \alpha$), the homogeneous retailer game leads to the highest expected revenue.

These results may justify the auctioneer policy of inviting non-retailers to the auction (not only more bidders but a different type of bidders) by organizing pre-sale exhibitions and editing luxury catalogues.

6. Conclusion

The games analysed in the present paper cast some light on the consequences for the strategy of a category of regular bidders (retailers) when they face a category of non-regular bidders (consumers). Notice that having consumers with a specific single-unit demand as opponents instead of other retailers leads a retailer to bid on average more aggressively, as long as the distribution of consumer valuations is not stochastically dominated.

The second-price sealed-bid auction with heterogeneous bidders is still equivalent to an English auction and efficient, since each object is attributed to the one who values it most. For that reason when two objects are sold, the sequence of second-price and Vickrey auctions generates equivalent revenues. The optimality question remains open. Myerson (1981) notices that in the case of a single auction with asymmetric bidders, the English auction is not optimal from the point of view of the seller, thus raising a dilemma between efficiency and optimality.[10]

The price trend computations for the heterogeneous game show that prices may decrease on average, which is an interesting finding and confirms that heterogeneous bidders may provide an explanation for the price decline anomaly. The magnitude of the decrease diminishes when consumers have, on average, higher valuations and/or the number of opponents increases, because the retailer's probability of winning diminishes despite the lower shading effect. Hence, the retailer's probability of losing both auctions increases and leads to an almost constant price trend.

When there are two bidders and low stochastic dominance of the consumers' valuations, the revenue of the seller is always greater for the heterogeneous game than for the homogeneous games. When there are more than two bidders at each

10 Maskin and Riley (1989) extend Myerson's (1981) analysis of optimal auctions to the case in which buyers have downward-sloping demand curves, independently drawn from one-parameter distribution, for quantities of a homogeneous good. They provide one of a number of expositions of revenue equivalence for the multi-unit case, when each buyer wants no more than a single unit.

auction, this is still true for moderate numbers of bidders and mild stochastic dominance.

Possible extensions to this work include introduction of optimal reserve prices and comparison with other forms of sequential auctions. Introduction of an optimal reserve price in our setting of heterogeneous bidders would be the same as solving an auction with three types of bidder, as the reserve price set by the seller will act as an extra bidder. Moreover, equilibrium in first-price auctions with decreasing marginal utility remains a tedious problem to solve. In addition, Maskin and Riley (2000) show that bidder asymmetry in single-object auctions results in an ambiguous ranking of first- and second-price auctions in terms of revenue generation. It is likely that such a result would be found in a comparison of sequential first-and second-price auctions, although a general ranking is highly unlikely.

Introducing extra retailers to the heterogeneous game turns out to be rather difficult and could be compared to solving a game with homogeneous bidders against a random reserve price. In effect, the strategy of a retailer in the first auction is more complex, as it depends on the profit he expects in the second auction, which is now conditional on the information revealed in the first auction. Several retailers now compete against each other in the first auction, and the winning retailer will also participate in the second auction. Consequently, the strategy of each retailer in the first auction depends on both the consumers' and the retailers' strategies in the first and second auctions. To solve this remains a challenge for future research.

Appendix A

Proof of proposition 1.
Let us consider a retailer with a pair of valuations (w^H, w^L). If he wins the first auction, then his valuation for the second object is equal to w^L, and it is a (weakly) dominant strategy in the second auction to bid w^L. If the retailer loses the first auction, his valuation for the second object is equal to w^H, and it is a (weakly) dominant strategy in the second auction to bid w^H.

Accordingly, one can compute the profits of the retailer in either case. If the retailer loses the first auction, his optimal expected profit in the second one is

$$\pi_\ell^B = \int_{\underline{v}}^{w^H} (w^H - s) dF^{c_2}(s) = \int_{\underline{v}}^{w^H} F^{c_2}(s)\, ds. \tag{A1}$$

Similarly, if the retailer wins the first auction, his expected profit in the second one is

$$\pi_w^B = \int_{\underline{v}}^{w^L} (w^L - s) dF^{c_2}(s) = \int_{\underline{v}}^{w^L} F^{c_2}(s)\, ds. \tag{A2}$$

If the retailer bids b in the first auction, his expected profit for the entire game is:

$$\int_{\underline{v}}^{b} (w^H - s) dF^{c_1}(s) + F^{c_1}(b) \int_{\underline{v}}^{w^L} F^{c_2}(s)\, ds + [1 - F^{c_1}(b)] \int_{\underline{v}}^{w^H} F^{c_2}(s)\, ds$$
$$= \int_{\underline{v}}^{b} \left(w^H - s - \int_{w^L}^{w^H} F^{c_2}(x)\, dx \right) dF^{c_1}(s) + \int_{\underline{v}}^{w^H} F^{c_2}(s)\, ds. \tag{A3}$$

The first-order condition yields at the optimum

$$b^B(w^H, w^L) = w^H - \int_{w^L}^{w^H} F^{c_2}(s)\, ds \tag{A4}$$

The second-order condition shows that this is a maximum. ■

Proof of result 1.
In each auction, if the retailer submits a bid b when competing against N identical consumers, the expected winning price is given by

$$E(b, N) = \int_{\underline{v}}^{b} s dF^N(s) + \int_{b}^{\bar{v}} \left(\int_{\underline{v}}^{s} \max(x, b) \frac{dF^{N-1}(x)}{F^{N-1}(s)} \right) dF^N(s)$$
$$= \int_{\underline{v}}^{b} s dF^N(s) + \int_{b}^{\bar{v}} \left(b \frac{F^{N-1}(b)}{F^{N-1}(s)} + \int_{b}^{s} x \frac{dF^{N-1}(x)}{F^{N-1}(s)} \right) dF^N(s)$$
$$= \int_{\underline{v}}^{\bar{v}} s dF^N(s) - N \int_{b}^{\bar{v}} \int_{b}^{s} F^{N-1}(x)\, dx dF(s)$$
$$= \int_{\underline{v}}^{\bar{v}} s dF^N(s) - N \int_{b}^{\bar{v}} F^{N-1}(s) [1 - F(s)]\, ds \tag{A5}$$

Notice that

$$E'(b, N) \equiv \frac{dE(b, N)}{db} = N F^{N-1}(b) [1 - F(b)] \tag{A6}$$

We can then rewrite $E(b, N)$ as

$$E(b, N) = \int_{\underline{v}}^{b} E'(s, N) ds + E(\underline{v}, N), \tag{A7}$$

where $E(\underline{v}, N) = N \int_{\underline{v}}^{\bar{v}} s [1 - F(s)]\, dF^{N-1}(s)$.

According to proposition 1, the retailer with valuations $w = (w^H, w^L)$ bids in the first auction $b^B \in (w^H, w^L)$ and bids in the second auction w^H if he lost in the first auction and w^L if he won the first auction. The expected difference between the winning price in the second auction and the winning price in the first auction is then given by

$$\begin{aligned}
E[p^2 - p^1|(w^H, w^L)] &= F^{c_1}(b^B)E(w^L, c_2) \\
&\quad + [1 - F^{c_1}(b^B)]E(w^H, c_2) - E(b^B, c_1) \\
&= -F^{c_1}(b^B)[E(w^H, c_2) - E(w^L, c_2)] \\
&\quad + [E(w^H, c_2) - E(b^B, c_1)] \\
&= -F^{c_1}(b^B)\left[c_2 \int_{w^L}^{w^H} F^{c_2-1}(s)[1 - F(s)]ds\right] \\
&\quad + \int_{\underline{v}}^{b^B} \left[c_2 F^{c_2-1}(s) - c_1 F^{c_1-1}(s)\right][1 - F(s)]ds \\
&\quad + c_2 \int_{b^B}^{w^H} F^{c_2-1}(s)[1 - F(s)]ds + [E(\underline{v}, c_2) - E(\underline{v}, c_1)] \\
&= -F^{c_1}(b^B)\int_{w^L}^{w^H} E'(s, c_2)ds - \int_{\underline{v}}^{b^B} E'(s, c_1)ds \\
&\quad + \int_{\underline{v}}^{w^H} E'(s, c_2)ds + [E(\underline{v}, c_2) - E(\underline{v}, c_1)]. \qquad \text{(A8)}
\end{aligned}$$

If we assume that $c_1 = c_2 = N$, then we have

$$E[p^2 - p^1|(w^H, w^L)] = \int_{b^B}^{w^H} E'(s, N)ds - F^N(b^B)\int_{w^L}^{w^H} E'(s, N)ds. \qquad \text{(A9)}$$

■

Proof of result 2.
The cumulative distribution of w^H is given by $G^2(\cdot)$, while the conditional distribution of w^L, given w^H, is $G(\cdot)/G(w^H)$. Hence we have

$$\begin{aligned}
E[b^B(w^H, w^L)] &= \int_{\underline{v}}^{\bar{v}} \left[\int_{\underline{v}}^{w^H} b^B(w^H, w^L)\frac{dG(w^L)}{G(w^H)}\right] dG^2(w^H) \\
&= \int_{\underline{v}}^{\bar{v}} \left[\int_{\underline{v}}^{w^H} w^H - \int_{w^L}^{w^H} F^{c_2}(s)ds\frac{dG(w^L)}{G(w^H)}\right] dG^2(w^H) \\
&= \int_{\underline{v}}^{\bar{v}} \left(w^H - \int_{\underline{v}}^{w^H} \frac{F^{c_2}(s)G(s)}{G(w^H)}ds\right) dG^2(w^H) \\
&= \left(\int_{\underline{v}}^{\bar{v}} w^H dG^2(w^H)\right) - 2\int_{\underline{v}}^{\bar{v}}\int_{\underline{v}}^{w^H} F^{c_2}(s)G(s)\,ds dG(w^H) \\
&= \int_{\underline{v}}^{\bar{v}} s dG^2(s) - 2\int_{\underline{v}}^{\bar{v}} F^{c_2}(s)G(s)[1 - G(s)]ds. \qquad \text{(A10)}
\end{aligned}$$

■

Proof of result 3.
We proceed as in the proof of result 2:

$$\begin{aligned}
E[b^K(w^H)] &= \int_{\underline{v}}^{\overline{v}} b^K(w^H) dG^2(w^H) \\
&= \int_{\underline{v}}^{\overline{v}} \left[w^H - \int_{\underline{v}}^{w^H} \frac{G^{2k-1}(s)}{G^{2k-1}(w^H)} ds \right] dG^2(w^H) \\
&= \int_{\underline{v}}^{\overline{v}} w^H dG^2(w^H) - \int_{\underline{v}}^{\overline{v}} \left[\int_{\underline{v}}^{w^H} G^{2k-1}(s) ds \right] \frac{dG^2(w^H)}{G^{2k-1}(w^H)} \\
&= \int_{\underline{v}}^{\overline{v}} w^H dG^2(w^H) - 2 \int_{\underline{v}}^{\overline{v}} \left[\int_{\underline{v}}^{w^H} G^{2k-1}(s) ds \right] G^{2-2k}(w^H) dG(w^H) \\
&= \int_{\underline{v}}^{\overline{v}} s dG^2(s) - \frac{2}{2k-3} \int_{\underline{v}}^{\overline{v}} [G^2(s) - G^{2k-1}(s)] ds. \qquad \text{(A11)}
\end{aligned}$$

■

Proof of result 4.
We need to show that $E[b^B] - E[b^K] \geq 0$ for $c_2 = k = N$ opponents.
⋆ Consider first that $F = G$ (no stochastic dominance):

$$\Rightarrow -2 \int_{\underline{v}}^{\overline{v}} \left\{ G^{N+1}(s)[1 - G(s)] - \frac{1}{2N-3} [G^2(s) - G^{2N-1}(s)] \right\} ds \geq 0. \qquad \text{(A12)}$$

We need to show that, $\forall N \geq 1$, $\forall s \in [\underline{v}, \overline{v}]$:

$$G^{N+1}(s)[1 - G(s)] - \frac{1}{2N-3} [G^2(s) - G^{2N-1}(s)] \leq 0. \qquad \text{(A13)}$$

- For $N = 1$, this is obvious for any distribution compatible with our assumptions, since this is true $\forall s \in [v, \overline{v}]$.
- $\forall N > 1$, we have $2N - 3 > 0$. By denoting $G(s) \in [0, 1]$ by x, we need to show that

$$\begin{aligned}
x^{N+1}(1-x) - \frac{1}{2N-3}(x^2 - x^{2N-1}) &\leq 0; \quad \forall x \in [0, 1], \forall N \geq 2 \\
x^2 \left[x^{N-1} - x^N - \frac{1}{2N-3}(1 - x^{2N-3}) \right] &\leq 0 \\
\frac{1 - x^{2N-3}}{(1-x)\, x^{N-1}} &\geq 2N - 3 \\
\frac{1 - x^{2N-3}}{1-x} \frac{1}{x^{N-1}} &\geq 2N - 3. \qquad \text{(A14)}
\end{aligned}$$

$$\text{Note that } \lim_{x\to 0} \frac{1-x^{2N-3}}{1-x}\frac{1}{x^{N-1}} = \infty > 2N-3$$
$$\text{and that } \lim_{x\to 1} \frac{1-x^{2N-3}}{1-x}\frac{1}{x^{N-1}} = 2N-3. \tag{A15}$$

Since $G(s)$ is strictly increasing between $G(\underline{v}) = 0$ and $G(\bar{v}) = 1$, inequality (A13) is verified $\forall s$, and thus inequality (A12) $\Rightarrow E[b^B] \geq E[b^K]$.

⋆ Consider now that F dominates G, i.e. $G(s) \geq F(s) \, \forall s \in [\underline{v}, \bar{v}]$. We must show that $F^N(s)G(s)[1-G(s)] - [G^2(s) - G^{2N-1}(s)]/(2N-3) \leq 0$. As we proved that the inequality holds $\forall s$ for $F(s) = G(s)$, the fact that $G^N(s) \geq F^N(s) \, \forall s$ and that the first term is non-negative $\forall s$ guarantees that this inequality also holds for $G(s) \geq F(s)$. ■

Appendix B

The expected revenues of the seller used in figure 2 for the homogeneous games, for various degrees of stochastic dominance (α) and competition (N, the number of opponent bidders), are obtained according to the following formulae when $G = s$ and $F = s^\alpha$, $\underline{v} = 0$ and $\bar{v} = 1$ (details on request). Denote by $F_k^{(n)}$ the distribution of $X_k^{(n)}$, the kth highest value among n independent draws from F.

Homogeneous consumer game

- $E(p^1) = \int_0^1 s dF_2^{(c_1)}(s) = \alpha^2 c_1(c_1 - 1)/(\alpha c_1 + 1)(\alpha c_1 - \alpha + 1)$.
- $E(p^2) = \int_0^1 s dF_2^{(c_2)}(s) = \alpha^2 c_2(c_2 - 1)/(\alpha c_2 + 1)(\alpha c_2 - \alpha + 1)$.

If $c_1 = c_2 = N + 1$, then $E(p^1) + E(p^2) = 2N\alpha^2(N+1)/(N\alpha + 1)(N\alpha + \alpha + 1)$

Homogeneous retailer game

The three outcomes of the price trend are summarized below for $k = N$ opponents.

We have $E[b^K({w_3}^{(2N+2)})] = E[{w_4}^{(2N+2)}] = (2N-1)/(2N+3)$, $E[b^K(w_2^{(2N+2)})] = (2N-1)(2N+1)/2N(2N+3)$ and $E[{w_3}^{(2N+2)}] = 2N/(2N+3)$ and thus

- $E(p^1) = (4N^2 + 2N - 2)/(2N+1)(2N+3)$.
- $E(p^2) = (4N^2 + 2N - 1)/(2N+1)(2N+3)$.

Thus, $E(p^1) + E(p^2) = [4N(2N+1) - 3]/(2N+1)(2N+3)$.

TABLE 1

The three possible outcomes of the price trend for $N+1$ retailers	p^1	p^2	Probability
i) The same retailer has the two highest valuations.	$b^K(w_3{}^{(2N+2)})$	$w_3{}^{(2N+2)}$	$\frac{1}{(2N+1)}$
ii) Different retailers have the two highest valuations and the bidder with the 2nd highest valuation:			
- does not have the 3rd highest valuation.	$b^K(w_2{}^{(2N+2)})$	$w_3{}^{(2N+2)}$	$\frac{(2N-1)}{(2N+1)}$
- also has the 3rd highest valuation.	$b^K(w_2{}^{(2N+2)})$	$w_4{}^{(2N+2)}$	$\frac{1}{(2N+1)}$

REFERENCES

Ashenfelter, O. (1989) ‘How auctions work for wine and art,’ *Journal of Economic Perspectives* 3, 23–36

Ashenfelter, O., and D. Genesove (1992) ‘Testing for price anomalies in real-estate auctions,’ *American Economic Review* 82, 501–5

Beggs, A., and K. Graddy (1997) ‘Declining values and the afternoon effect: evidence from art auctions,’ *RAND Journal of Economics* 28, 544–65

Black, J., and D. de Meza (1992) ‘Systematic price differences between successive auctions are no anomaly,’ *Journal of Economics and Management Strategy* 1, 607–28

Burguet, R., and J. Sakovics (1997) ‘Sequential auctions with supply or demand uncertainty,’ *Revista Española de Economia* 14, 23–40

Branco, F. (1997) ‘Sequential auctions with synergies: an example,’ *Economics Letters* 54, 159–63

Caillaud, B., and C. Mezzetti (2004) ‘Equilibrium reserve prices in sequential ascending auctions,’ *Journal of Economic Theory* 117, 78–95

Cassady, R. (1967) *Auctions and Auctioneering* (Berkeley: University of California Press)

Chanel, O., L.-A. Gérard-Varet, and S. Vincent (1996) ‘Auction theory and practice: evidence from the market of jewellery,’ in *Economics of the Arts: Selected Essays*, ed. V. Ginsburgh and P.-M. Menger (Amsterdam: North-Holland)

Chanel, O., and S. Vincent (2004) ‘Computing price trend in sequential auctions,’ *Louvain Economic Review* 70, 443–60

Engelbrecht-Wiggans, R., and R. Weber (1993) ‘A sequential auction involving asymmetrically-informed bidders,’ *International Journal of Game Theory* 12, 123–27

von der Fehr, N.-H.M. (1994) ‘Predatory bidding in sequential auctions,’ *Oxford Economic Papers* 46, 345–56

Gale, I.L., and D.B. Hausch (1994) ‘Bottom fishing and declining prices in sequential auctions,’ *Games and Economic Behavior* 7, 318–31

Hausch, D. (1986) ‘Multi-object auctions: sequential vs. simultaneous sales?’ *Management Science* 32, 1599–610

Ginsburgh V. (1998) ‘Absentee bidders and the declining price anomaly in wine auctions,’ *Journal of Political Economy* 106, 1302–19

Katzman, B. (1999) ‘A two stage sequential auction with multi-unit demands,’ *Journal of Economic Theory* 86, 77–99

Kittsteiner T., J. Nikutta, and E. Winter (2004) ‘Declining valuations in sequential auctions,’ *International Journal of Game Theory* 33, 89–106

Krishna, V., and R. Rosenthal (1996) 'Simultaneous auctions with synergies,' *Games and Economic Behavior* 17, 1–31

McAfee, R.P., and D. Vincent (1993) 'The declining price anomaly,' *Journal of Economic Theory* 60, 191–212

Maskin, E.S., and J.G. Riley (1989) 'Optimal multi-unit auctions,' in *The Economics of Missing Markets, Information, and Games*, ed. F. Hahn (Oxford: Oxford University Press)

— (2000) 'Asymmetric auctions,' *Review of Economic Studies* 67, 413–38

Menezes, F.M., and P.K. Monteiro (2004) 'Auctions with synergies and asymmetric buyers,' *Economic Letters* 85, 287–94

Milgrom, P., and R. Weber (2000) 'A theory of auctions and competitive bidding, II,' in *The Economic Theory of Auctions*, ed. P. Klemperer (Cambridge: Edward Elgar)

Myerson, R. (1981) 'Optimal auction design,' *Mathematics of Operations Research* 6, 58–73

Ortega Reichert, A. (1968) 'A sequential game with information flow,' chap. 8, Ph.D thesis, Stanford University, reprinted in *The Economic Theory of Auctions*, ed. P. Klemperer (Cambridge: Edward Elgar)

Pesando, J.E., and P.M. Shum (1996) 'Price anomalies at auction: evidence from the market for modern prints,' in *Economics of the Arts: Selected Essays*, ed. V. Ginsburgh and P.-M. Menger (Amsterdam: North-Holland)

Pezanis-Christou, P. (1997) 'Three essays on competitive bidding,' PhD thesis, European University Institute

Rosenthal, R., and R. Wang (1996) 'Simultaneous auctions with synergies and common values,' *Games and Economic Behavior* 17, 32–55

Van den Berg, G, J.C. Van Ours, and M.P. Pradhan (2001) 'The declining price anomaly in Dutch Dutch rose auctions,' *American Economic Review* 91, 1055–62

Vincent, S. (1998) 'Sequential auctions with heterogeneous bidders,' Discussion Paper 98-19, University of Copenhagen, Centre for Industrial Economics

Weber, R. (1983) 'Multiple object auctions,' in *Auction Bidding and Contracting: Uses and Theory*, ed. R. Engelbrecht-Wiggans, M. Shubik, and R. Stark (New York: New York University Press)